PROOF OF FORMULA 4.253.1

$$\int_0^1 x^{\mu-1}(1-x^r)^{\nu-1}\ln x\,dx = \frac{1}{r^2}B\left(\frac{\mu}{r},\nu\right)\left[\psi\left(\frac{\mu}{r}\right)-\psi\left(\frac{\mu}{r}+\nu\right)\right]$$

Differentiate the integral representation

$$B(a,b) = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)} = \int_0^1 t^{a-1}(1-t)^{b-1}\,dt$$

to obtain

$$\int_0^1 t^{a-1}(1-t)^{b-1}\ln t\,dt = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}\left(\psi(a)-\psi(a+b)\right).$$

Replace t by x^r and let $c = ar$ to obtain

$$\int_0^1 x^{c-1}(1-x^r)^{b-1}\ln x\,dx = \frac{\Gamma(c/r)\Gamma(b)}{r^2\Gamma(a/r+b)}\left[\psi\left(\frac{c}{r}\right)-\psi\left(\frac{c}{r}+b\right)\right].$$

This is the result after replacing c by μ and b by ν.