PROOF OF FORMULA 4.253.5

$$\int_1^\infty (x-1)^{p-1} \ln x \, dx = \frac{\pi}{p \,\sin \pi p}$$

Let $t = 1/x$ to obtain

$$\int_1^\infty (x-1)^{p-1} \ln x \, dx = -\int_0^1 t^{-1-p}(1-t)^{p-1} \, \ln t \, dt.$$

Entry 4.253.2 states that

$$\int_0^1 x^{a-1}(1-x)^{-a-1} \, \ln x \, dx = -\frac{\pi}{a \sin \pi a},$$

and usng it with $a = -p$ gives the result.