

PROOF OF FORMULA 4.253.7

$$\int_0^\infty \frac{\ln x \, dx}{(a+x)^{n+\frac{1}{2}}} = \frac{2}{(2n-1)a^{n-\frac{1}{2}}} \left[\ln a + 2 \ln 2 - 2 \sum_{k=1}^{n-1} \frac{1}{2k-1} \right]$$

Entry 4.253.6 states that

$$\int_0^\infty \frac{\ln x \, dx}{(a+x)^{p+1}} = \frac{1}{pa^p} (\ln a - \gamma - \psi(p))$$

The result follows by putting $p = n - \frac{1}{2}$ and using entry 8.366.3

$$\psi\left(n - \frac{1}{2}\right) = -\gamma + 2 \sum_{k=1}^{n-1} \frac{1}{2k-1} - 2 \ln 2.$$