PROOF OF FORMULA 4.255.2

$$\int_0^1 \ln x \, \frac{(1+x^2)x^{p-2}}{1-x^{2p}} \, dx = -\left(\frac{\pi}{2p}\right)^2 \sec^2 \frac{\pi}{2p}$$

Let $t = x^{2p}$ to obtain

$$\int_0^1 \ln x \, \frac{(1+x^2)x^{p-2}}{1-x^{2p}} \, dx = \frac{1}{4p^2} \int_0^1 \frac{t^{a-1}+t^{-a}}{1-t} \, \ln t \, dt$$

with $a = (p-1)/2p$.

The basic representation

$$\psi(x) = \int_0^1 \left(-\frac{1}{\ln t} - \frac{t^{x-1}}{1-t} \right) dt$$

gives

$$\int_0^1 \frac{t^{a-1}+t^{-a}}{1-t} \, \ln t \, dt = -\left[\psi'(a) + \psi'(1-a) \right].$$

The result follows from the identity

$$\psi'(a) + \psi'(1-a) = \frac{\pi^2}{\sin^2 \pi a}.$$