PROOF OF FORMULA 4.267.16

$$\int_0^1 \frac{x^{p-1} - x^{q-1}}{(1+x^r)\ln x}\, dx = \ln\left[\frac{\Gamma\left(\frac{p+r}{2r}\right)\Gamma\left(\frac{q}{2r}\right)}{\Gamma\left(\frac{q+r}{2r}\right)\Gamma\left(\frac{p}{2r}\right)}\right]$$

The change of variables $y = x^r$ yields

$$\int_0^1 \frac{x^{p-1} - x^{q-1}}{(1+x^r)\ln x}\, dx = \int_0^1 \frac{y^{p/r-1} - y^{q/r-1}}{(1+y)\ln y}\, dy.$$

This integral appears as entry 4.267.9.