PROOF OF FORMULA 4.267.17

$$\int_0^1 \frac{(1 - x^{2p-2q})}{1 + x^{2p}} \frac{x^{q-1}\,dx}{\ln x} = \ln \tan \frac{\pi q}{4p}$$

The change of variables $t = x^{2p}$ gives

$$\int_0^1 \frac{(1 - x^{2p-2q})}{1 + x^{2p}} \frac{x^{q-1}\,dx}{\ln x} = \int_0^1 \frac{t^{r-1} - t^{-r}}{(1+t)\,\ln t}\,dt,$$

with $r = q/2p$. The result now follows from entry 4.267.10

$$\int_0^1 \frac{x^{p-1} - x^{-p}}{(1+x)\,\ln x}\,dx = \ln \tan \frac{\pi p}{2}.$$