PROOF OF FORMULA 4.267.19

$$\int_0^\infty \frac{x^{p-1}-x^{q-1}}{(1-x^r)\,\ln x}\,dx = \ln\left[\sin\frac{\pi p}{r}\,\operatorname{cosec}\frac{\pi q}{r}\right]$$

The change of variables $x = e^t$ gives

$$\int_0^\infty \frac{x^{p-1}-x^{q-1}}{(1-x^r)\,\ln x}\,dx = \int_{-\infty}^\infty \frac{e^{pt}-e^{qt}}{1-e^{rt}}\,\frac{dt}{t}.$$

This integral appears as entry 3.411.30. The result follows from there.