PROOF OF FORMULA 4.267.30

$$\begin{aligned}\int_0^\infty \frac{(1-x^p)(1-x^q)x^{s-1}\,dx}{(1-x^{p+2q+2s})\,\ln x} &= 2\int_0^1 \frac{(1-x^p)(1-x^q)x^{s-1}\,dx}{(1-x^{p+2q+2s})\,\ln x} \\ &= 2\ln\left[\sin\left(\frac{\pi s}{p+q+2s}\right)\ \operatorname{cosec}\left(\frac{(p+s)\pi}{p+q+2s}\right)\right]\end{aligned}$$

The first identity follows by separating the integral on $[0,1]$ and $[1,\infty)$ and letting $x \mapsto 1/x$ in the second integral. To evaluate the integral, write it as

$$\int_0^\infty \frac{(1-x^p)(1-x^q)x^{s-1}\,dx}{(1-x^{p+2q+2s})\,\ln x} = \int_0^\infty \frac{x^{s-1}-x^{q+s-1}}{(1-x^{p+q+2s})\,\ln x}\,dx - \int_0^\infty \frac{x^{p+s-1}-x^{p+q+s-1}}{(1-x^{p+q+2s})\,\ln x}\,dx.$$

Each of these integrals is evaluated by using entry 4.267.23

$$\int_0^\infty \frac{x^{p-1}-x^{q-1}}{(1-x^r)\,\ln x}\,dx = \ln\left[\frac{\sin\frac{\pi p}{r}}{\sin\frac{\pi q}{r}}\right].$$