PROOF OF FORMULA 4.272.13

$$\int_0^1 \frac{x^q - x^{-q}}{1-x^2} \left(\ln \frac{1}{x} \right)^p dx = \Gamma(p+1) \sum_{k=1}^{\infty} \left\{ \frac{1}{(2k+q-1)^{p+1}} - \frac{1}{(2k-q-1)^{p+1}} \right\}$$

Start with the expansion

$$\frac{1}{1-x^2} = \sum_{k=0}^{\infty} x^{2k}$$

to obtain

$$\int_0^1 \frac{x^q - x^{-q}}{1-x^2} \left(\ln \frac{1}{x} \right)^p dx = \sum_{k=0}^{\infty} \int_0^1 x^{q+2k} \left(\ln \frac{1}{x} \right)^p dx - \sum_{k=0}^{\infty} \int_0^1 x^{-q+2k} \left(\ln \frac{1}{x} \right)^p dx.$$

Now use

$$\int_0^1 x^a \left(\ln \frac{1}{x} \right)^b dx = \int_0^{\infty} t^b e^{-(a+1)t} \, dt = \frac{\Gamma(b+1)}{(a+1)^{b+1}}$$

to obtain the result.