PROOF OF FORMULA 4.272.16

$$\int_0^1 \left(\ln \frac{1}{x} \right)^n (1-x^q)^m \, x^{p-1} \, dx = n! \sum_{k=0}^{m} \binom{m}{k} \frac{(-1)^k}{(p+kq)^{n+1}}$$

The change of variables $t = \ln 1/x$ yields

$$\int_0^1 \left(\ln \frac{1}{x} \right)^n (1-x^q)^m \, x^{p-1} \, dx = \int_0^\infty t^n e^{-pt} (1-e^{-qt})^m \, dt.$$

Expand the binomial term to obtain

$$\int_0^1 \left(\ln \frac{1}{x} \right)^n (1-x^q)^m \, x^{p-1} \, dx = \sum_{k=0}^{m} (-1)^k \binom{m}{k} \int_0^\infty t^n e^{-t(p+kq)} \, dt.$$

The result now follows from the change of variables $v = t(p+kq)$ and the value

$$\int_0^\infty v^n e^{-v} dv = \Gamma(n+1) = n!.$$