

FORMULA 4.272.19

$$\begin{aligned} \int_0^1 \left( \ln \frac{1}{x} \right)^{2n-1} \frac{x^p - x^{-p}}{1 - x^q} x^{q-1} dx &= \frac{1}{p^{2n}} \sum_{k=n}^{\infty} \left( \frac{2\pi p}{q} \right)^k \frac{|B_{2k}|}{2k(2k-2n)!} \\ &= \frac{\Gamma(2n)}{q^{2n}} \left[ \zeta \left( 2n, \frac{p}{q} \right) - \zeta \left( 2n, -\frac{p}{q} \right) \right] \end{aligned}$$