PROOF OF FORMULA 4.273

$$\int_u^v \left(\ln \frac{x}{u} \right)^{p-1} \left(\ln \frac{v}{x} \right)^{q-1} \frac{dx}{x} = B(p,q) \left(\ln \frac{v}{u} \right)^{p+q-1}$$

Let $x = ut$ to obtain

$$\int_u^v \left(\ln \frac{x}{u} \right)^{p-1} \left(\ln \frac{v}{x} \right)^{q-1} \frac{dx}{x} = \int_1^c \ln^{p-1} t \, (\ln c - \ln t)^{q-1} \frac{dt}{t}$$

with $c = v/u$. The change of variable $y = \ln t$ gives

$$\int_u^v \left(\ln \frac{x}{u} \right)^{p-1} \left(\ln \frac{v}{x} \right)^{q-1} \frac{dx}{x} = \int_0^{\ln c} y^{p-1} (\ln c - y)^{q-1} \, dy.$$

Now let $z = y/\ln c$ to obtain the result.