

PROOF OF FORMULA 4.275.1

$$\int_0^1 \left[\left(\ln \frac{1}{x} \right)^{q-1} - x^{p-1} (1-x)^{q-1} \right] dx = \frac{\Gamma(q)}{\Gamma(p-q)} [\Gamma(p+q) - \Gamma(p)]$$

The change of variables $t = -\ln x$ gives

$$\int_0^1 \left(\ln \frac{1}{x} \right)^{q-1} dx = \int_0^\infty t^{q-1} e^{-t} dt = \Gamma(q).$$

Therefore the integral is

$$\Gamma(q) - B(p, q) = \frac{\Gamma(q)}{\Gamma(p-q)} [\Gamma(p+q) - \Gamma(p)]$$

as stated.