

PROOF OF FORMULA 4.281.4

$$\int_0^1 \left(\frac{1}{\ln x} + \frac{x^{a-1}}{1-x} \right) dx = -\psi(a)$$

Entry 3.427.1 states that

$$\psi(a) = \int_0^\infty \left(\frac{e^{-t}}{t} - \frac{e^{-at}}{1-e^{-t}} \right) dt.$$

Let $x = e^{-t}$ to obtain the result.