PROOF OF FORMULA 4.291.13

$$\int_0^\infty \frac{\ln(1+x)}{x(1+x)}\,dx = \frac{\pi^2}{6}$$

The change of variables in the integral over $[1, \infty)$ gives

$$\int_1^\infty \frac{\ln(1+x)}{x(1+x)}\,dx = \int_0^1 \frac{\ln(1+x)}{1+x}\,dx - \int_0^1 \frac{\ln x}{1+x}\,dx.$$

It follows that

$$\int_0^\infty \frac{\ln(1+x)}{x(1+x)}\,dx = \int_0^1 \frac{\ln(1+x)}{x}\,dx - \int_0^1 \frac{\ln x}{1+x}\,dx.$$

Entry 4.291.1 gives

$$\int_0^1 \frac{\ln(1+x)}{x}\,dx = \frac{\pi^2}{12}$$

and entry 4.271.2 gives

$$\int_0^1 \frac{\ln x}{1+x}\,dx = -\frac{\pi^2}{12}.$$

This gives the result.