

**PROOF OF FORMULA 4.291.15**

$$\int_0^\infty \frac{\ln(1+x)}{(ax+b)^2} dx = \frac{\ln a - \ln b}{a(a-b)}$$

Integrate by parts to produce

$$\int_0^\infty \frac{\ln(1+x)}{(ax+b)^2} dx = \frac{1}{a} \int_0^\infty \frac{dx}{(1+x)(ax+b)}.$$

The partial fraction decomposition

$$\frac{1}{(1+x)(ax+b)} = \frac{1}{b-a} \frac{1}{1+x} - \frac{a}{b-a} \frac{1}{ax+b}$$

is used to integrate from 0 to  $N$  to produce

$$\int_0^\infty \frac{\ln(1+x)}{(ax+b)^2} dx = \lim_{N \rightarrow \infty} \frac{1}{a(b-a)} (\ln(1+N) - \ln(aN+b) + \ln b).$$

This gives the result.