PROOF OF FORMULA 4.291.2

$$\int_0^1 \frac{\ln(1-x)}{x}dx = -\frac{\pi^2}{6}$$

Expand the integrand as

$$\ln(1-x) = -\sum_{k=1}^{\infty} \frac{x^k}{k}$$

from which one gets

$$\frac{\ln(1-x)}{x} = -\sum_{k=0}^{\infty} \frac{x^k}{k+1}.$$

Integrate term-by-term to produce

$$\int_0^1 \frac{\ln(1-x)}{x}dx = -\sum_{k=0}^{\infty} \frac{1}{(k+1)^2}.$$

That is the result.