

**PROOF OF FORMULA 4.291.21**

$$\int_0^{\infty} \frac{\ln(ax+b) dx}{(1+x)^2} = \frac{a \ln a - b \ln b}{a-b}$$

The change of variables  $t = bx$  gives

$$\int_0^{\infty} \frac{\ln(ax+b) dx}{(1+x)^2} = b \ln \frac{a}{b} \int_0^{\infty} \frac{dt}{(t+b)^2} + b \int_0^{\infty} \frac{\ln(t+c) dt}{(t+b)^2},$$

with  $c = b^2/a$ . The first integral is elementary and the second one appears as entry 4.291.17

$$\int_0^{\infty} \frac{\ln(a+x) dx}{(x+b)^2} = \frac{a \ln a - b \ln b}{b(a-b)}.$$