PROOF OF FORMULA 4.291.3

$$\int_0^{1/2} \frac{\ln(1-x)}{x} dx = \frac{1}{2}\ln^2 2 - \frac{\pi^2}{12}$$

Integration by parts gives

$$\int_0^{1/2} \frac{\ln(1-x)}{x} dx = \ln^2 2 + \int_0^{1/2} \frac{\ln x}{1-x}\, dx.$$

The change of variable $t = 1 - x$ yields

$$\int_0^{1/2} \frac{\ln(1-x)}{x} dx = \ln^2 2 + \int_{1/2}^{1} \frac{\ln(1-t)}{t}\, dt.$$

Now use

$$\int_0^{1/2} \frac{\ln(1-x)}{x} dx + \int_{1/2}^{1} \frac{\ln(1-x)}{x} dx = \int_0^{1} \frac{\ln(1-x)}{x} dx = -\frac{\pi^2}{6}$$

according to 4.291.2. The result follows from here.