PROOF OF FORMULA 4.292.1

$$\int_0^1 \frac{\ln(1 \pm x)}{\sqrt{1 - x^2}}\, dx = -\frac{\pi}{2}\ln 2 \pm 2G$$

The change of variable $x = \sin t$ and symmetry give

$$\int_0^1 \frac{\ln(1 \pm x)}{\sqrt{1 - x^2}}\, dx = \int_0^{\pi/2} \ln(1 \pm \sin t)\, dt = \frac{1}{2}\int_0^\pi \ln(1 \pm \sin t)\, dt.$$

The result now follows from entry 4.224.10 that gives

$$\int_0^\pi \ln(1 \pm \sin t)\, dt = -\pi \ln 2 \pm 4G.$$