PROOF OF FORMULA 4.292.3

$$\int_{-a}^{a} \frac{\ln(1+bx)}{\sqrt{a^2-x^2}}\,dx = \pi \ln\left[\frac{1+\sqrt{1-a^2b^2}}{2}\right]$$

Let $x = at$ to obtain, with $c = ab$,

$$\int_{-a}^{a} \frac{\ln(1+bx)}{\sqrt{a^2-x^2}}\,dx = \int_{-1}^{1} \frac{\ln(1+ct)}{\sqrt{1-t^2}}\,dt.$$

The change of variables $t = \sin\varphi$, followed by $\theta = \pi/2 + \varphi$ yields

$$\int_{-a}^{a} \frac{\ln(1+bx)}{\sqrt{a^2-x^2}}\,dx = \int_{0}^{\pi} \ln(1-c\cos\theta)\,d\theta.$$

This integral appears as entry 4.224.12 giving the result.