PROOF OF FORMULA 4.293.10

$$\int_0^\infty x^{\mu-1} \ln(1 + bx)\, dx = \frac{\pi}{\mu b^\mu \, \sin \pi \mu}$$

The change of variable $t = bx$ gives

$$\int_0^\infty x^{\mu-1} \ln(1 + bx)\, dx = b^{-\mu} \int_0^\infty t^{\mu-1} \ln(1 + t)\, dt.$$

The result now follows from entry 4.293.3:

$$\int_0^\infty t^{\mu-1} \ln(1 + t)\, dt = \frac{\pi}{\mu \, \sin \pi \mu}.$$

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