## PROOF OF FORMULA 4.293.12

$$\int_0^1 \frac{\ln(1+x)\,dx}{(1+x)^{\mu+1}} = -\frac{\ln 2}{\mu 2^\mu} + \frac{2^\mu - 1}{\mu^2\, 2^\mu}$$

Start with the identity

$$\int_0^1 (1+x)^{\nu-\mu-1}\,dx = \frac{2^{\nu-\mu}-1}{\nu-\mu}.$$

Differentiate with respect to $\nu$ to obtain

$$\int_0^1 (1+x)^{\nu-\mu-1}\ln(1+x)\,dx = \frac{1}{(\nu-\mu)^2}\left(2^{\nu-\mu}(\nu-\mu)\ln 2 - (2^{\nu-\mu}-1)\right).$$

The special case $\nu = 0$ gives the result.