PROOF OF FORMULA 4.293.14

$$\int_0^\infty \frac{x^{\mu-1}\,\ln(x+a)}{(x+a)^\nu}\,dx = a^{\mu-\nu}B(\mu,\nu-\mu)\left[\psi(\nu)-\psi(\nu-\mu)+\ln a\right]$$

The integral representation

$$B(\mu,\nu) = \int_0^\infty \frac{t^{\mu-1}\,dt}{(1+t)^{\mu+\nu}}$$

gives

$$\int_0^\infty \frac{x^{\mu-1}\,dx}{(a+x)^{\mu+\nu}} = a^{\mu-\gamma}B(\mu,\nu-\mu)$$

The result follows by differentiating with respect to ν and using the formula

$$\frac{d}{d\nu}B(\mu,\nu) = B(\mu,\nu)\left[\psi(\nu)-\psi(\nu+\mu)\right].$$