PROOF OF FORMULA 4.293.6

$$\int_0^1 x^{n-1/2}\ln(1+x)\,dx = \frac{2\ln 2}{2n+1} + \frac{4(-1)^n}{2n+1}\left[\frac{\pi}{4} - \sum_{j=0}^{n}\frac{(-1)^j}{2j+1}\right]$$

Entry 4.293.1 states that

$$\int_0^1 x^{\mu-1}\ln(1+x)\,dx = \frac{1}{\mu}\left[\ln 2 - \beta(\mu+1)\right].$$

The case discussed here corresponds to $\mu = n + \frac{1}{2}$. Thus

$$\int_0^1 x^{n-1/2}\ln(1+x)\,dx = \frac{2}{2n+1}\left[\ln 2 - \beta(n+\tfrac{3}{2})\right].$$

To simplify the answer observe that

$$\begin{aligned}
\beta(n+\tfrac{3}{2}) &= \sum_{k=0}^{\infty}\frac{(-1)^k}{n+3/2+k} \\
&= 2\sum_{j=n+1}^{\infty}\frac{(-1)^{j-n-1}}{2j+1} \\
&= 2(-1)^{n+1}\left(\sum_{j=0}^{\infty}\frac{(-1)^j}{2j+1} - \sum_{j=0}^{n}\frac{(-1)^j}{2j+1}\right).
\end{aligned}$$

The result follows from the classical evaluation

$$\sum_{j=0}^{\infty}\frac{(-1)^j}{2j+1} = \frac{\pi}{4}.$$