PROOF OF FORMULA 4.293.7

$$\int_0^\infty x^{\mu-1} \ln|1-x|\, dx = \frac{\pi}{\mu} \cot \pi\mu$$

The integral is

$$\int_0^\infty x^{\mu-1} \ln|1-x|\, dx = \int_0^1 x^{\mu-1} \ln(1-x)\, dx + \int_1^\infty x^{\mu-1} \ln(x-1)\, dx.$$

Entry 4.293.8 shows that the first integral is $-(\psi(\mu+1)-\gamma)/\mu$ and entry 4.293.9 gives the second one as $(\pi \cot \pi\mu + \psi(\mu+1) - \gamma)/\mu$. This gives the result.