PROOF OF FORMULA 4.293.9

$$\int_1^\infty x^{\mu-1} \, \ln(x-1) \, dx = \frac{1}{\mu} \left[\pi \cot \pi\mu + \psi(\mu+1) + \gamma \right]$$

Let $t = 1/x$ to obtain

$$\int_1^\infty x^{\mu-1} \, \ln(x-1) \, dx = \int_0^1 t^{-\mu-1} \ln(1-t) \, dt - \int_0^1 t^{-\mu-1} \ln t \, dt.$$

The first integral is $(\psi(1-\mu)+\gamma)/\mu$ by entry 4.293.8. To evaluate the second integral let $t = e^{-y}$ to obtain

$$\int_0^1 t^{-\mu-1} \ln t \, dt = \int_0^\infty y e^{\mu y} \, dy = -\frac{1}{\mu^2},$$

after integrating by parts. The result now follows from the identity

$$\psi(1-\mu) = \psi(\mu) + \pi \cot \pi\mu.$$