PROOF OF FORMULA 4.297.1

$$\int_0^1 \ln\left(\frac{ax+b}{bx+a}\right) \frac{dx}{(1+x)^2} = \frac{1}{a-b}\left[(a+b)\ln\frac{a+b}{2} - a\ln a - b\ln b\right]$$

The integral is

$$\int_0^1 \ln\left(\frac{ax+b}{bx+a}\right) \frac{dx}{(1+x)^2} = \int_0^1 \frac{\ln(ax+b)\,dx}{(1+x)^2} - \int_0^1 \frac{\ln(bx+a)\,dx}{(1+x)^2}.$$

These integrals are evaluated using entry 4.291.20

$$\int_0^1 \frac{\ln(ax+b)\,dx}{(1+x)^2} = \frac{1}{2(a-b)}\left[(a+b)\ln(a+b) - b\ln b - a\ln 2\right]$$

to obtain the result.