

NEW FORMULA 4.322.2

The original formula is

$$\int_0^{\infty} \frac{\ln \sin^2 ax}{b^2 + x^2} dx = \frac{\pi}{b} \ln \frac{1 - e^{-2ab}}{2}$$

The change of variables $x = bt$ and replacing ab by a gives the new formula (going back to x as the integration variable)

$$\int_0^{\infty} \frac{\ln \sin^2 ax}{1 + x^2} dx = \pi \ln \frac{1 - e^{-2a}}{2}$$