PROOF OF FORMULA 4.325.11

$$\int_0^1 \frac{x^{\mu-1}}{\sqrt{\ln 1/x}} \ln\ln 1/x \, dx = -(\gamma + \ln 4\mu)\sqrt{\frac{\pi}{\mu}}$$

The change of variables $t = \ln 1/x$ gives

$$\int_0^1 \frac{x^{\mu-1}}{\sqrt{\ln 1/x}} \ln\ln 1/x \, dx = \int_0^\infty t^{-1/2} \, e^{-\mu t} \, \ln t \, dt.$$

Then $s = t^{1/2}$ yields

$$\int_0^\infty t^{-1/2} \, e^{-\mu t} \, \ln t \, dt = 4 \int_0^\infty e^{-\mu s^2} \, \ln s \, ds.$$

This last integral appears as entry 4.333 giving the result.