PROOF OF FORMULA 4.342.3

$$\int_0^\infty e^{-\mu x} \left[\ln(\sinh x) - \ln x \right] \, dx = \frac{1}{\mu} \left(\ln \frac{\mu}{2} - \frac{1}{\mu} - \psi \left(\frac{\mu}{2} \right) \right)$$

Integrate by parts to obtain

$$\int_0^\infty e^{-\mu x} \left[\ln(\sinh x) - \ln x \right] \, dx = \frac{1}{\mu} \int_0^\infty e^{-\mu x} \left(\coth x - \frac{1}{x} \right) \, dx.$$

Express the hyperbolic functoon in terms of exponentials to get

$$\int_0^\infty e^{-\mu x} \left[\ln(\sinh x) - \ln x \right] \, dx = -\frac{1}{\mu^2} - \frac{1}{\mu} \int_0^\infty e^{-\mu x} \left(\frac{1}{x} - \frac{2}{1 - e^{-2x}} \right) \, dx.$$

The change of variables $s = 2x$ and the integral representation 8.361.8

$$\psi(z) = \ln z + \int_0^\infty e^{-xz} \left(\frac{1}{x} - \frac{1}{1 - e^{-x}} \right) \, dx$$

give the result.