PROOF OF FORMULA 4.352.2

$$\int_0^\infty x^n e^{-\mu x} \ln x \, dx = \frac{n!}{\mu^{n+1}} \left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \gamma - \ln \mu \right]$$

Formula 4.352.1 states that

$$\int_0^\infty x^{\nu-1} e^{-\mu x} \ln x \, dx = \frac{\Gamma(\nu)}{\mu^\nu} \left(\psi(\nu) - \ln \mu \right).$$

The special case $\nu = n + 1$ and the value

$$\psi(n+1) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \gamma,$$

give the result.