PROOF OF FORMULA 4.353.1

$$\int_0^\infty (x-\nu)x^{\nu-1}e^{-x}\ln x\,dx = \Gamma(\nu)$$

Expand to obtain

$$\int_0^\infty (x-\nu)x^{\nu-1}e^{-x}\ln x\,dx = \int_0^\infty x^{nu}e^{-x}\ln x\,dx - \nu\int_0^\infty x^{\nu-1}e^{-x}\ln x\,dx.$$

Using formula 4.352.1:

$$\int_0^\infty x^{\nu-1}e^{-\mu x}\ln x\,dx = \frac{\Gamma(\nu)}{\mu^\nu}\left(\psi(\nu)-\ln\mu\right),$$

produces

$$\int_0^\infty (x-\nu)x^{\nu-1}e^{-x}\ln x\,dx = \Gamma(\nu+1)\psi(\nu+1) - \nu\Gamma(\nu)\psi(\nu).$$

The result simplifies using

$$\Gamma(\nu+1) = \nu\Gamma(\nu) \text{ and } \psi(\nu+1) = \psi(\nu)+\frac{1}{\nu}.$$