PROOF OF FORMULA 4.353.2

$$\int_0^\infty (\mu x - n - \tfrac{1}{2}) x^{n-1/2} e^{-\mu x} \, \ln x \, dx = \frac{(2n-1)!!}{(2\mu)^n} \sqrt{\frac{\pi}{\mu}}$$

Start with

$$\begin{aligned} \int_0^\infty (\mu x - n - \tfrac{1}{2}) x^{n-1/2} e^{-\mu x} \, \ln x \, dx &= \mu \int_0^\infty x^{n+1/2} e^{-\mu x} \, \ln x \, dx \\ &- (n + \tfrac{1}{2}) \int_0^\infty x^{n-1/2} e^{-\mu x} \, \ln x \, dx. \end{aligned}$$

The result now follows from 4.352.3:

$$\int_0^\infty x^{m-1/2} e^{-\mu x} \, \ln x \, dx = \frac{\sqrt{\pi}(2m-1)!!}{2^m \mu^{m+1/2}} \left[2 \left(1 + \tfrac{1}{3} + \cdots + \tfrac{1}{2m-1} \right) - \gamma - \ln 4\mu \right],$$

and some elementary simplifications.