PROOF OF FORMULA 4.353.3

$$\int_0^1 (\mu x + n + 1) x^n e^{\mu x} \ln x \, dx = e^{\mu} \sum_{k=0}^{n} \frac{(-1)^{k-1} \, n!}{(n-k)! \mu^{k+1}} + (-1)^n \frac{n!}{\mu^{n+1}}$$

Observe that

$$\frac{d}{dx} \left(e^{\mu x} x^{n+1} \right) = (\mu x + n + 1) x^n e^{\mu x}.$$

Integration by parts gives

$$\int_0^1 (\mu x + n + 1) x^n e^{\mu x} \ln x \, dx = - \int_0^1 x^n e^{\mu x} \, dx.$$

The result now follows from entry 3.351.1.