PROOF OF FORMULA 4.355.1

$$\int_0^\infty x^2 e^{-\mu x^2} \ln x \, dx = \frac{1}{8\mu}(2 - \ln 4\mu - \gamma)\sqrt{\frac{\pi}{\mu}}$$

Start with the identity

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} \, dx$$

and let $x = t^b$ and then $t = \mu^{1/b} y$ to produce

$$\int_0^\infty y^{ab-1} e^{-\mu y^b} \, dy = \frac{1}{b\mu^a}\Gamma(a).$$

Introduce $r = ab - 1$ to obtain

$$\int_0^\infty y^r e^{-\mu y^b} \, dy = \frac{1}{b}\mu^{-(r+1)/b}\Gamma\left(\frac{r+1}{b}\right).$$

Differentiate with respect to r to obtain

$$\int_0^\infty y^r e^{-\mu y^b} \ln y \, dy = \frac{\Gamma(p)}{b^2]\mu^p}\left[\psi(p) - \ln \mu\right]$$

where $p = (r+1)/b$.

The special case $r = b = 2$ gives $p = 3/2$. The value

$$\psi\left(\frac{3}{2}\right) = 2 + \psi\left(\frac{1}{2}\right) = 2 - 2\ln 2 - \gamma$$

gives the result.