PROOF OF FORMULA 4.358.5

$$\int_0^\infty x^{\nu-1} e^{-\mu x} (\ln x)^n \, dx = \frac{\partial^n}{\partial \nu^n} \left[\mu^{-\nu} \Gamma(\nu) \right]$$

Start with the representation

$$\Gamma(\nu) = \int_0^\infty t^{\nu-1} e^{-t} \, dt = \mu^\nu \int_0^\infty x^{\nu-1} e^{-\mu x} \, dx$$

to produce

$$\mu^{-\nu} \Gamma(\nu) = \int_0^\infty x^{\nu-1} e^{-\mu x} \, dx.$$

Now differentiate n times with respect to ν.