

PROOF OF FORMULA 4.369.1

$$\int_0^{\infty} x^{\nu-1} e^{-\mu x} [\psi(\nu) - \ln x] dx = \frac{\Gamma(\nu) \ln \mu}{\mu^{\nu}}$$

The integral is

$$\int_0^{\infty} x^{\nu-1} e^{-\mu x} [\psi(\nu) - \ln x] dx = \psi(\nu) \int_0^{\infty} x^{\nu-1} e^{-\mu x} dx - \int_0^{\infty} x^{\nu-1} e^{-\mu x} \ln x dx.$$

The first integral gives $\Gamma(\nu)/\mu^{\nu}$ by a simple scaling. The second one appears as entry 4.352.1 as

$$\int_0^{\infty} x^{\nu-1} e^{-\mu x} \ln x dx = \frac{\Gamma(\nu)}{\mu^{\nu}} [\psi(\nu) - \ln \mu].$$

This gives the result.