PROOF OF FORMULA 6.281.1

$$\int_0^\infty [1 - \text{erf}(px)]\, x^{2q-1}\, dx = \frac{\Gamma(q+1/2)}{2\sqrt{\pi} q p^{2q}}$$

Let $t = px$ to obtain

$$\int_0^\infty [1 - \text{erf}(px)]\, x^{2q-1}\, dx = \frac{1}{p^{2q}} \int_0^\infty [1 - \text{erf}(t)]\, t^{2q-1}\, dt.$$

The error function is defined by

$$\text{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-s^2}\, ds,$$

so it satisfies

$$\frac{d}{dt}\text{erf}(t) = \frac{2}{\sqrt{\pi}} e^{-t^2}.$$

Integration by parts gives

$$\int_0^\infty [1 - \text{erf}(px)]\, x^{2q-1}\, dx = \frac{1}{q p^{2q} \sqrt{\pi}} \int_0^\infty t^{2q} e^{-t^2}\, dt.$$

The change of variables $y = t^2$ shows that the last integral is $\Gamma(q+1/2)$.