

PROOF OF FORMULA 6.282.1

$$\int_0^{\infty} \operatorname{erf}(qx)e^{-px} dx = \frac{1}{p} \left[1 - \operatorname{erf} \left(\frac{p}{2q} \right) \right] e^{p^2/4q^2}$$

Let $s = qx$ and integrate by parts to obtain

$$\begin{aligned} \int_0^{\infty} \operatorname{erf}(qx)e^{-px} dx &= \frac{2}{p\sqrt{\pi}} \int_0^{\infty} e^{-ps/q-s^2} ds \\ &= \frac{2}{p\sqrt{\pi}} e^{p^2/4q^2} \int_0^{\infty} e^{-(s+p/2q)^2} ds \\ &= \frac{2}{p\sqrt{\pi}} e^{p^2/4q^2} \int_{p/2q}^{\infty} e^{-s^2} ds \end{aligned}$$

and this is the result.