

PROOF OF FORMULA 8.259.1

$$\int_{-\infty}^{\infty} e^{-px^2} \Phi(a + bx) dx = \sqrt{\frac{\pi}{p}} \Phi\left(\frac{a\sqrt{p}}{\sqrt{b^2 + p}}\right)$$

Recall that the error function is given by

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt,$$

so it satisfies $\Phi'(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$.

Let $\lambda(a)$ be the integral and differentiate to get

$$\lambda'(a) = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-(p+b^2)x^2 - 2abx - a^2} dx.$$

Complete the square to integrate and obtain

$$\lambda'(a) = \frac{1}{\sqrt{p + b^2}} e^{-a^2/(p^2 + b^2)}.$$

Now integrate with respect to a to obtain the result.