ENTRIES IN PART 6

3.191.1 $$\int_0^a x^{\nu-1}(a-x)^{\mu-1}\,dx = a^{\mu+\nu-1}B(\mu,\nu)$$

3.191.2 $$\int_a^\infty x^{-\nu}(x-a)^{\mu-1}\,dx = a^{\mu-\nu}B(\nu-\mu,\mu)$$

3.191.3 $$\int_0^1 x^{\nu-1}(1-x)^{\mu-1}\,dx = \int_0^1 x^{\mu-1}(1-x)^{\nu-1}\,dx = B(\mu,\nu)$$

3.192.1 $$\int_0^1 \frac{x^p\,dx}{(1-x)^p} = \frac{\pi p}{\sin \pi p}$$

3.192.2 $$\int_0^1 \frac{x^p\,dx}{(1-x)^{p+1}} = -\frac{\pi p}{\sin \pi p}$$

3.192.3 $$\int_0^1 \frac{(1-x)^p\,dx}{x^{p+1}} = -\frac{\pi p}{\sin \pi p}$$

3.192.4 $$\int_1^\infty (x-1)^{p-1/2}\frac{dx}{x} = \frac{\pi}{\cos \pi p}$$

3.193 $$\int_0^n x^{\nu-1}(n-x)^n\,dx = \frac{n!\,n^{\nu+n}}{\nu(\nu+1)(\nu+2)\cdots(\nu+n)}$$

3.194.3 $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+bx)^\nu} = b^{-\mu}B(\mu,\nu-\mu)$$

3.194.4 $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+bx)^{n+1}} = \frac{(-1)^n}{\sin \pi\mu}\frac{\pi}{b^\mu}\binom{\mu-1}{n}$$

3.194.6 $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(1+bx)^2} = \frac{(1-\mu)\pi}{b^\mu\,\sin \pi\mu}$$

3.194.7 $$\int_0^\infty \frac{x^m\,dx}{(a+bx)^{n+1/2}} = 2^{m+1}m!\frac{(2n-2m-3)!!}{(2n-1)!!}\,\frac{a^{m-n+1/2}}{b^{m+1}}$$

3.196.2 $$\int_a^\infty (x+b)^{-\nu}(x-a)^{\mu-1}\,dx = (a+b)^{\mu-\nu}B(\nu-\mu,\mu)$$

3.196.3 $$\int_a^b (x-a)^{\nu-1}(b-x)^{\nu-1}\,dx = (b-a)^{\mu+\nu-1}B(\mu,\nu)$$

3.196.4 $$\int_1^\infty \frac{dx}{(a-bx)(x-1)^\nu} = -\frac{\pi}{b\,\sin\pi\nu}(1-a/b)^{-\nu}$$

3.196.5 $$\int_{-\infty}^1 \frac{dx}{(a-bx)(1-x)^\nu} = \frac{\pi}{b\,\sin\pi\nu}(a/b-1)^{-\nu}$$

3.216.1 $$\int_0^1 \frac{x^{\mu-1}+x^{\nu-1}}{(1+x)^{\mu+\nu}}\,dx = B(\mu,\nu)$$

3.216.2 $$\int_1^\infty \frac{x^{\mu-1}+x^{\nu-1}}{(1+x)^{\mu+\nu}}\,dx = B(\mu,\nu)$$

3.217 $$\int_0^\infty \left(\frac{b^p x^{p-1}}{(1+bx)^p} - \frac{(1+bx)^{p-1}}{b^{p-1}x^p}\right)dx = \pi\cot\pi p$$

3.218 $$\int_0^\infty \frac{x^{2p-1}-(a+x)^{2p-1}}{(a+x)^p\,x^p}\,dx = \pi\cot\pi p$$

3.221.1 $$\int_a^\infty \frac{(x-a)^{p-1}}{x-b}\,dx = \frac{\pi(a-b)^{p-1}}{\sin\pi p}$$

3.222.2 $$\int_0^\infty \frac{x^{\mu-1}\,dx}{x+a} = \frac{\pi\,a^{\mu-1}}{\sin\pi\mu} \text{ if } a>0$$
$$= -\frac{\pi\,(-a)^{\mu-1}}{\tan\pi\mu} \text{ if } a<0$$

3.223.1 $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(b+x)(a+x)} = -\frac{\pi}{\sin\pi\mu}\,\frac{b^{\mu-1}-a^{\mu-1}}{b-a}$$

3.223.2 $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(b+x)(a-x)} = \frac{\pi}{a+b}\left(\frac{b^{\mu-1}}{\sin\pi\mu}+\frac{a^{\mu-1}}{\tan\pi\mu}\right)$$

3.223.3 $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(a-x)(b-x)} = -\frac{\pi}{\tan\pi\mu}\frac{a^{\mu-1}-b^{\mu-1}}{a-b}$$

3.224 $$\int_0^\infty \frac{(x+b)x^{\mu-1}\,dx}{(x+a)(x+c)} = \frac{\pi}{\sin\pi\mu}\left(\frac{a-b}{a-c}a^{\mu-1}+\frac{c-b}{c-a}c^{\mu-1}\right)$$

3.225.1 $$\int_1^\infty \frac{(x-1)^{p-1}\,dx}{x^2} = \frac{\pi(1-p)}{\sin \pi p}$$

3.225.2 $$\int_1^\infty \frac{(x-1)^{1-p}\,dx}{x^3} = \frac{\pi p(p-1)}{2\sin \pi p}$$

3.225.3 $$\int_0^\infty \frac{x^p\,dx}{(1+x)^3} = \frac{\pi p(1-p)}{2\sin \pi p}$$

3.226.1 $$\int_0^1 \frac{x^n\,dx}{\sqrt{1-x}} = \frac{2\,(2n)!!}{(2n+1)!!}$$

3.226.2 $$\int_0^1 \frac{x^{n-1/2}\,dx}{\sqrt{1-x}} = \frac{(2n-1)!!}{(2n)!!}\,\pi$$

3.241.2 $$\int_0^\infty \frac{x^{\mu-1}\,dx}{1+x^\nu} = \frac{\pi}{\nu}\frac{1}{\sin(\pi\mu/\nu)} = \frac{1}{\nu}B\left(\frac{\mu}{\nu}, 1-\frac{\mu}{\nu}\right)$$

3.241.4 $$\int_0^\infty \frac{x^{\mu-1}\,dx}{(p+qx^\nu)^{n+1}} = \frac{1}{\nu p^{n+1}}\left(\frac{p}{q}\right)^{\mu/\nu} B\left(\frac{\mu}{\nu}, n+1-\frac{\mu}{\nu}\right)$$

3.241.5 $$\int_0^\infty \frac{x^{p-1}\,dx}{(1+x^q)^2} = \frac{(p-q)\pi}{q^2}\operatorname{cosec}\frac{(p-q)\pi}{q}$$

3.248.1 $$\int_0^\infty \frac{x^{\mu-1}\,dx}{\sqrt{1+x^\nu}} = \frac{1}{\nu}B\left(\frac{\mu}{\nu}, \frac{1}{2}-\frac{\mu}{\nu}\right)$$

3.248.2 $$\int_0^1 \frac{x^{2n+1}\,dx}{\sqrt{1-x^2}} = \frac{(2n)!!}{(2n+1)!!}$$

3.248.3 $$\int_0^1 \frac{x^{2n}\,dx}{\sqrt{1-x^2}} = \frac{(2n-1)!!}{(2n)!!}\,\frac{\pi}{2}$$

3.249.1 $$\int_0^\infty \frac{dx}{(x^2+a^2)^n} = \frac{(2n-3)!!}{2(2n-2)!!}\,\frac{\pi}{a^{2n-1}}$$

3.249.2 $$\int_0^a (a^2-x^2)^{n-1/2}\,dx = a^{2n}\frac{(2n-1)!!\,\pi}{2(2n)!!}$$

3.249.5 $$\int_0^1 (1-x^2)^{\mu-1}\,dx = 2^{2\mu-2}B(\mu,\mu) = \tfrac{1}{2}B\left(\tfrac{1}{2},\mu\right)$$

3.249.7 $$\int_0^1 (1-x^\mu)^{-1/\nu}\,dx = \frac{1}{\mu}B\left(\frac{1}{\mu}, 1-\frac{1}{\nu}\right)$$

3.249.8 $$\int_{-\infty}^\infty \left(1+\frac{x^2}{n-1}\right)^{-n/2} dx = \frac{\sqrt{\pi(n-1)}}{\Gamma(n/2)}\Gamma\left(\frac{n-1}{2}\right)$$

3.251.1 $$\int_0^1 x^{\mu-1}(1-x^\lambda)^{\nu-1}\,dx = \frac{1}{\lambda}B\left(\frac{\mu}{\lambda},\nu\right)$$

3.251.2 $$\int_0^\infty x^{\mu-1}(1+x^2)^{\nu-1}\,dx = \frac{1}{2}B\left(\frac{\mu}{2},1-\nu-\frac{\mu}{2}\right)$$

3.251.3 $$\int_1^\infty x^{\mu-1}(x^p-1)^{\nu-1}\,dx = \frac{1}{p}B\left(1-\nu-\frac{\mu}{p},\nu\right)$$

3.251.4 $$\int_0^\infty \frac{x^{2m}\,dx}{(ax^2+c)^n} = \frac{(2m-1)!!\,(2n-2m-3)!!\,\pi}{2(2n-2)!!\,a^m c^{n-m-1}\sqrt{ac}}$$

3.251.5 $$\int_0^\infty \frac{x^{2m+1}\,dx}{(ax^2+c)^n} = \frac{m!\,(n-m-2)!}{2(n-1)!\,a^{m+1}c^{n-m-1}}$$

3.251.6 $$\int_0^\infty \frac{x^{\mu+1}\,dx}{(1+x^2)^2} = \frac{\mu\pi}{4\sin(\pi\mu/2)}$$

3.251.8 $$\int_0^1 x^{q+p-1}(1-x^q)^{-p/q}\,dx = \frac{\pi p}{q^2\,\sin(\pi p/q)}$$

3.251.9 $$\int_0^1 x^{q/p-1}(1-x^q)^{-1/p}\,dx = \frac{\pi}{q\,\sin(\pi/p)}$$

3.251.10 $$\int_0^1 x^{p-1}(1-x^q)^{-p/q}\,dx = \frac{\pi}{q\,\sin(\pi p/q)}$$

3.251.11 $$\int_0^\infty x^{\mu-1}(1+bx^p)^{-\nu}\,dx = \frac{1}{p}b^{-\mu/p}B\left(\frac{\mu}{p},\nu-\frac{\mu}{p}\right)$$

3.267.1 $$\int_0^1 \frac{x^{3n}\,dx}{\sqrt[3]{1-x^3}} = \frac{2\pi}{3\sqrt{3}}\,\frac{\Gamma(n+\frac{1}{3})}{\Gamma(\frac{1}{3})\,\Gamma(n+1)}$$

3.267.2 $$\int_0^1 \frac{x^{3n-1}\,dx}{\sqrt[3]{1-x^3}} = \frac{(n-1)!\,\Gamma(\frac{2}{3})}{3\Gamma(n+\frac{2}{3})}$$

3.267.3 $$\int_0^1 \frac{x^{3n-2}\,dx}{\sqrt[3]{1-x^3}} = \frac{\Gamma(n-\frac{1}{3})\,\Gamma(\frac{2}{3})}{3\Gamma(n+\frac{1}{3})}$$

3.311.3 $$\int_{-\infty}^{\infty} \frac{e^{-px}\,dx}{1-e^{-qx}} = \frac{\pi}{|q|}\text{cosec }\frac{p\pi}{q}$$

3.311.9 $$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{b+e^{-x}} = \frac{\pi\, b^{\mu-1}}{\sin\pi\mu}$$

3.312.1 $$\int_{0}^{\infty} (1-e^{-x/b})^{\nu-1}\, e^{-\mu x}\,dx = bB(b\mu,\nu)$$

3.313.1 $$\text{PV}\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{1-e^{-x}} = \pi\,\cot\pi\mu$$

3.313.2 $$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{(1+e^{-x})^{\nu}} = B\left(\mu,\nu-\mu\right)$$

3.314 $$\int_{-\infty}^{\infty} \frac{e^{\mu x}\,dx}{(e^{b/c}+e^{-x/c})^{\nu}} = c\exp\left[b(\mu-\nu/c)\right]B\left(c\mu,\nu-c\mu\right)$$

3.457.3 $$\int_{-\infty}^{\infty} \frac{x\,dx}{(a^2e^{x}+e^{-x})^{\mu}} = -\frac{1}{2a^{\mu}}B\left(\frac{\mu}{2},\frac{\mu}{2}\right)\ln a$$

4.251.1 $$\int_{0}^{\infty} \frac{x^{\mu-1}\,\ln x\,dx}{b+x} = \frac{\pi b^{\mu-1}}{\sin\pi\mu}\left(\ln b-\pi\cot\pi\mu\right)$$

4.273 $$\int_{u}^{v} \left(\ln\frac{x}{u}\right)^{p-1}\left(\ln\frac{v}{x}\right)^{q-1}\frac{dx}{x} = B(p,q)\left(\frac{v}{u}\right)^{p+q-1}$$

4.275.1 $$\int_{0}^{1} \left[(\ln 1/x)^{q-1}-x^{p-1}(1-x)^{q-1}\right]dx = \frac{\Gamma(q)}{\Gamma(p+q)}\left[\Gamma(p+q)-\Gamma(p)\right]$$

4.321.1 $$\int_{-\infty}^{\infty} x\,\ln\cosh x\,dx = 0$$