MATH 115, FALL SEMESTER 2008

Exam 1

Name (print): _____

INSTRUCTIONS

(1) Calculators are allowed.
(2) Present your solutions in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

QN	PTS
1	
2	
3	
4	
5	
TOTAL	

(1) The value of a radio after t years is give by $V(t) = 30 - 4t$, where the value is in dollars.

(a) (6 points) Find the x-intercept and y-intercept.

Solution. The y–intercept is $V(0) = 30$.
The x–intercept is the point where $V(x) = 0$, thus it is 7.5. □

(b) (6 points) Interpret the intercepts in dollars and years.

Solution. The y–intercept means that the value of the radio is \$30 at the beginning.
The x–intercept means that after 7.5 years, the radio is worthless. □

(c) (7 points) After two years, another person is willing to pay \$18 for the radio. Should you take the offer? Explain clearly.

Solution. After two years, the value of the radio is $V(2) = 30 - 4 \cdot 2 = 22$ dollars. The person is only willing to pay \$18, so it is a bad deal. □

(2) Given the functions $f(x), g(x), h(x)$,

x	$f(x)$	$g(x)$	$h(x)$
0	0.5	2.5	0.5
1	1	1	1
2	1.5	0.4	2
3	2	0.16	4
4	2.5	0.064	8

(a) (7 points) Find which function(s) is/are linear, which is/are exponential. Explain clearly.

Solution. $f(x+1) - f(x) = 0.5, g(x+1)/g(x) = 0.4, h(x+1)/h(x) = 2$, so $f(x)$ is linear, $g(x)$ and $h(x)$ are exponential. □

(b) (7 points) Is $g(x)$ concave up or concave down? Explain clearly.

Solution. Concave up, because the function is decreasing slower (or in other words, growing faster). □

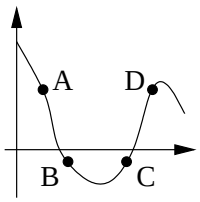
(c) (7 points) Find the average rate of change for $g(x)$ between $x = 0$ to 4.

Solution. The average rates of change are:

$$\frac{g(4) - g(0)}{4 - 0} = \frac{0.064 - 2.5}{4} = \frac{-2.436}{4} = -0.609$$

□

(3) Given the graph below, find out at which point the function is



(a) (3 points) increasing and concave up.

Solution. C □

(b) (3 points) increasing and concave down.

Solution. D □

(c) (3 points) decreasing and concave up.

Solution. B □

(d) (3 points) decreasing and concave down.

Solution. A □

(4) A company makes shoes. It has a fixed cost of \$3,000, and variable cost of \$5 for each pair of shoes. Its revenue function is $R(q) = 20q$.

(a) (7 points) What is the cost function?

Solution. The cost function is

$$C(q) = 3000 + 5q$$

□

(b) (7 points) How much is the company selling for each pair of shoes?

Solution. \$20. □

(c) (8 points) How many pairs does the company have to sell to break even?.

Solution. To break even, we must have $C(q) = R(q)$. So $3000 + 5q = 20q$, which means $q = 200$. The company has to sell 200 pairs to break even. □

(5) A lab is growing a certain kind of bacteria in solution. Due to negligence, there is lack of nutrition in the solution. As a result the population of the bacteria is decreasing at the rate of 10% per hour. (Round to nearest hundredth)

(a) (10 points) What is the *continuous* growth rate?

Solution. The function for the population is

$$P(t) = P_0(1 - 10\%)^t = P_0 0.9^t = P_0 e^{t \ln 0.9},$$

therefore the continuous growth rate is $\ln 0.9 \approx -0.11$. □

(b) (10 points) What is the half-life of the population?

Solution. Solving $0.5 = 0.9^t$, we get $\ln 0.5 = \ln 0.9^t = t \ln 0.9$. Hence the half-life is

$$\frac{\ln 0.5}{\ln 0.9} \approx 6.59(hr)$$

□

(c) (10 points) The error is found after 5 hours, and additional nutrition is added to the solution. The population grows at the rate of 20% thereafter. In how many hours after that can the population grow back to its original size?

Solution. After 5 hours, the population is $P(5) = P_0 0.9^5 = 0.59 P_0$. So the function for the population after 5 hours is $P(t) = 0.59 P_0 (1 + 0.2)^t$. Solving

$$0.59 P_0 1.2^t = P_0,$$

we take natural log on both sides,

$$\ln\left(0.59 \cdot 1.2^t\right) = \ln 1 = 0,$$

$$\ln 0.59 + t \ln 1.2 = 0$$

$$t = -\frac{\ln 0.59}{\ln 1.2} \approx 2.89(hr)$$

□