MATH 115, FALL SEMESTER 2008

Exam 3

Name (print): ____________________________

INSTRUCTIONS

(1) Calculators are allowed.
(2) Present your solutions in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

QN	PTS
1	
2	
3	
4	
5	
TOTAL	

(1) Find derivatives.

(a) (10 points) $f(x) = x^2 e^{3x}$

Solution. $f'(x) = 2xe^{3x} + 3x^2 e^{3x} = (2x + 3x^2)e^{3x}$ □

(b) (10 points) $g(x) = 3e^{2x} - 4\ln(2x) + 5\sqrt{2x}$

Solution. $g'(x) = 3 \cdot 2e^{2x} - 4\frac{2}{2x} + 5\frac{2}{2\sqrt{2x}} = 6e^{2x} - \frac{8}{2x} + \frac{5}{\sqrt{2x}}$ □

(c) (10 points) $h(x) = \ln\sqrt{x}$

Solution. $h'(x) = \frac{1}{\sqrt{x}}\frac{1}{2\sqrt{x}} = \frac{1}{2x}$ □

(d) (10 points) $y(x) = \frac{x+1}{\ln x}$

Solution. $y'(x) = \frac{\ln x - (x+1)(\frac{1}{x})}{(\ln x)^2}$ □

(2) (10 points) Find the equation to the tangent line of the function at the given point.

$$y = \frac{x}{x+1}e^x, \quad (0,0)$$

Solution.

$$y' = \frac{(e^x + xe^x)(x+1) - xe^x}{(x+1)^2} = \frac{e^x(x^2+x+1)}{(x+1)^2}$$

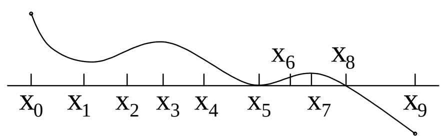
$$y'(0) = 1$$

Therefore the equation to the tangent line is

$$y - 0 = 1(x - 0)$$

□

(3) (9 points) Answer the questions based on the graph.



(a) (10 points) The graph is for the function $f(x)$. List the local maxima/minima, inflection points.

Solution. local maximum: x_0, x_3, x_7
local minimum: x_1, x_5, x_9
inflection point: x_2, x_4, x_6 □

(b) (6 points) The graph is for the function $f'(x)$. List the local maxima/minima, inflection points.

Solution. local maximum: x_8
local minimum: none
inflection point: x_1, x_3, x_5, x_7 □

(c) (4 points) The graph is for the function $f''(x)$. List the inflection points.

Solution. inflection point: x_8 □

(4) (10 points) Use the definition of derivative to prove that

$$(ax^2+bx+c)' = 2ax+b,$$

where a, b are constants. No credit for any other method.

Solution. Let $f(x) = ax^2+bx+c$.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{\big(a(x+h)^2+b(x+h)+c\big)-\big(ax^2+bx+c\big)}{h} \\
&= \lim_{h\to 0} \frac{\big(a(x^2+2hx+h^2)+(bx+hb)+c\big)-\big(ax^2+bx+c\big)}{h} \\
&= \lim_{h\to 0} \frac{\big(a(2hx+h^2)+bh\big)-\big(ax^2+bx+c\big)}{h} \\
&= \lim_{h\to 0} \left(a(2x+h)+b\right) \\
&= 2ax+b
\end{aligned}$$

□

(5) Find the critical points, local maxima/minima, inflection points for the functions given.

(a) (10 points) $y = x^5 - 15x^3 + 10$

Solution.

$$y' = 5x^4 - 45x^2 = 5x^2(x^2-9)$$

So the critical points are $x = 0, \pm 3$.
local minimum: $x = 3$.
local maximum: $x = -3$.
$y'' = 20x^3 - 90x = 10x(2x^2-9)$. The solutions are $x = 0, \pm\sqrt{9/2}$ and all of them are inflection points. □

(b) (10 points) $y = x^5 - 5x^4 - 5$

Solution.

$$y' = 5x^4 - 20x^3 = 5x^3(x-4)$$

So the critical points are $x = 0, 4$.
local minimum: $x = 4$.
local maximum: $x = 0$.
$y'' = 20x^3 - 60x^2 = 20x^2(x-3)$. The solutions are $x = 0, 3$ and only $x = 3$ are inflection points □