MATH 320, Spring SEMESTER 2008

Exam 2

Name (print): ____________________________
Signature: ____________________________

INSTRUCTIONS

(1) Calculators are NOT allowed to use.
(2) Do NOT simpilify the answers. Answers with factorials or powers are acceptable, e.g., 10!, $9 \times 8 \times 7$ or $100 - 2^4$.
(3) Present your solutions in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

QN	PTS
1	
2	
3	
4	
5	
6	
TOTAL	

(1) (10 points) In an 82-NHL season, how many different seasonal records are possible if a team can either win, lose, or tie each game? (For the seasonal record, we do not not care if the team wins or loses any single specific game)

Solution. Let x_1, x_2, x_3 be the number of games a team can win, lose, or tie, then

$$x_1 + x_2 + x_3 = 82, \quad x_1 \geq 0, \quad x_2 \geq 0, \quad x_2 \geq 0.$$

So the total number of possible records is

$$\binom{82+3-1}{82} = \binom{84}{82}$$

□

(2) (a) (10 points) How many different words can be formed using all the letters of the word "code-word"?

Solution. There are $c \times 1, o \times 2, d \times 2, e \times 1, w \times 1, r \times 1$, so the total number is $\binom{8}{1 \quad 2 \quad 2 \quad 1 \quad 1 \quad 1}$

□

(b) (10 points) What if those words are written on the edge of a quarter?

Solution. $\frac{1}{8}\binom{8}{1 \quad 2 \quad 2 \quad 1 \quad 1 \quad 1}$ □

(c) (10 points) What if in the first part, the sequence "ord" is not allowed? (not on the edge of a quarter)

Solution. Consider the words can be formed using c, o, d, e, w, ord. There are 6 distinct letters, so the total number of words formed is 6!. Therefore the number of words without "ord" is $\binom{8}{1 \quad 2 \quad 2 \quad 1 \quad 1 \quad 1} - 6!$ □

(3) (10 points) In how many different ways can we put 5 red and 3 blue rooks on a 8-by-8 board so that no two rooks can attack one another?

Solution. $8!\binom{8}{3}$ □

(4) (a) (10 points) Find the coefficient of $x^4y^3z^5$ in the expansion of $(3x - y - 2z)^9$.

Solution. 0 □

(b) (10 points) Find the coefficient of $x^3y^2z^4$ in the expansion of $(3x - y - 2z)^9$.

Solution. $3^3(-1)^2(-2)^4\begin{pmatrix} & 9 & \\ 3 & 2 & 4 \end{pmatrix}$ □

(5) (10 points) Prove that

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}$$

Solution. $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} = \binom{n-2}{k} + \binom{n-2}{k-1} + \binom{n-1}{k-1} = \binom{n-3}{k} + \binom{n-3}{k-1} + \binom{n-2}{k-1} + \binom{n-1}{k-1}$. □

(6) Consider the set $\{x|1 \leq x \leq 10\}$, we define the relation $\leq$ by $a \leq b$ if and only if there exists non-negative integer c such that $b = a + 3c$.

(a) (10 points) Prove that this relation is a partial order.

Solution. Note that $x \leq y$ if and only if x is less than or equals to y and $y - x$ is a multiple of 3.
$\leq$ is reflective because for any a, $a = a + 3 \cdot 0$.
$\leq$ is antisymmetric because if $x \neq y$ and $x = y + 3c$ with non-negative integer c, we cannot have $y = x + 3d$ with d non-negative.
$\leq$ is trasitive because if $x \leq y$ and $y \leq z$ then $z - a$ is a multiple of 3 too. □

(b) (5 points) Find the longest chain in the poset.

Solution. $1 \leq 4 \leq 7 \leq 10$ □

(c) (5 points) Find the largest set of antichain in the poset.

Solution. $\{1, 2, 3\}$ □