Group A: Use inclusion–exclusion to provide a combinatorial proof to the following.

$$\sum_{i=0}^{k}(-1)^i\binom{k}{i}(k-i)^n = \begin{cases} n! & \text{if } k=n, \\ 0 & \text{if } k>n. \end{cases}$$

Hint: Let S be a set of m elements. Then m^n is the number of ways to assign numbers $1,\ldots,n$ to the elements in S. In other words, m^n is the number of mappings from $\{1,\ldots,n\}$ to S.

Group B: –

Group C: Use inclusion–exclusion to provide a combinatorial proof to the following.

$$\sum_{i=0}^{n}(-1)^i\binom{n}{i}\binom{m+n-i}{k-i} = \begin{cases} \binom{m}{k} & \text{if } m\geq k, \\ 0 & \text{if } m<k. \end{cases}$$

Group D: Let $\mu(x)=\mu(1,x)$ be the Möbius function to the poset $(\mathbb{N},|)$. Determine

$$\sum_{n\leq x}\mu(n)\left\lfloor\frac{x}{n}\right\rfloor.$$