

P40, 20: Prove that $r(3, 3, 3) \leq 17$.

Proof. Suppose a graph has 17 vertices, whose edges are colored in red, blue or green. Pick any vertex x and there are 16 edges coming out of it. By Pigeonhole Principle, at least 6 of them have the same color, say red. If any two of these end points are connected by a red edge, then those two points and x forms a red triangle. Otherwise, we have 6 vertices whose edges are colored in 2 colors, blue and green. Since $r(3, 3) = 6$, we are guaranteed to have a blue or green triangle.

Therefore $r(3, 3, 3) \leq 17$. □

P40, 27: A collection of subsets of $\{1, 2, \dots, n\}$ has the property that each pair of subsets has at least one element in common. Prove that there are at most 2^{n-1} subsets in the collection.

Proof. Group a set and its compliment into pairs. Since there are a total of 2^n subsets, we have exactly 2^{n-1} pairs. If we choose more than 2^{n-1} subsets in the collection, at least two of them will be in the same pair. However, by the definition, those two subsets have no common element, which contradicts to the assumption given. □