

P75, 39: (1) $\binom{20}{6}$

(2) Let the positions chosen be x_1, \dots, x_6 , and $y_1 = x_1 - 0, y_2 = x_2 - x_1, \dots, y_5 = x_5 - x_4, y_6 = x_6 - x_5, y_7 = 20 - x_6$. So

$$\sum y_i = 20, \quad y_1 \geq 1, y_2, \dots, y_6 \geq 2, y_7 \geq 0.$$

Now let $z_1 = y_1 - 1, z_j = y_j - 2 \quad 2 \leq j \leq 6, z_7 = y_7$, then

$$\sum z_i = 9, z_i \geq 0.$$

Therefore the number of solutions is $\binom{9+7-1}{9}$.

(3) Same idea, but $y_j \geq 3$.

P75, 48: Consider $m + n + 1$ positions filled with $m + 1$ A 's, and n B 's.

The number of ways to fill the positions is equal to the number of permutations of m A 's and *at most* n B 's, because the B 's after the $(m + 1)^{th}$ A can be considered the extras. So the number of permutations is $\binom{m+n+1}{m+1}$.

P153, 12: When n is odd, $(-1)^k \binom{n}{k} + (-1)^{n-k} \binom{n}{n-k} = 0$, so the terms will cancel out.

When $n = 2m$, the coefficient of $x^n = x^{2m}$ in $(1-x^2)^n$ is $(-1)^m \binom{2m}{m}$, while the coefficient of x^n in $(1+x)^n(1-x)^n = (\sum_i \binom{n}{i} x^i) (\sum_i \binom{n}{i} (-x)^i)$ is $\sum_k \binom{n}{k} \binom{n}{n-k} (-1)^{n-k} = \sum_k (-1)^k \binom{n}{k}^2$.

P75, 15:

$$(1-x)^n = \sum_i \binom{n}{i} (-1)^i x^i$$

Take derivative on both sides, and set $x = 1$.