P153, 26: $\binom{2n+2}{n+1}$ is the number of ways to choose $n+1$ objects from $2n+2$ objects.

We can also choose those $n+1$ objects by considering the first $2n$ objects. There are three cases.

(1) All $n+1$ objects are in the first $2n$ objects. There are $\binom{2n}{n+1}$ ways to choose.

(2) There are n objects in the first $2n$ objects, and one in the last two. There are $\binom{2n}{n}\binom{2}{1}$ ways to choose.

(3) There are $n-1$ objects in the first $2n$ objects. There are $\binom{2n}{n-1} = \binom{2n}{n+1}$ ways to choose.

Extra: It is easy to see that it is true when $k=1$.

When $k \geq 2$, we have

$$\begin{aligned}
&\sum_{n_1+\cdots+n_k=n} \begin{pmatrix} & n & \\ n_1 & \cdots & n_k \end{pmatrix} x_1^{n_1} \cdots x_k^{n_k} \\
= & \sum_{n_1} \binom{n}{n_1} x_1^{n_1} \left(\sum_{n_2+\cdots+n_k=n-n_1} \begin{pmatrix} & n-n_1 & \\ n_2 & \cdots & n_k \end{pmatrix} x_2^{n_2} \cdots x_k^{n_k} \right) \\
= & \sum_{n_1} \binom{n}{n_1} x_1^{n_1} (x_2 + \cdots + x_k^{n_k})^{n-n_1} \\
= & (x_1 + \cdots + x_k^{n_k})^n.
\end{aligned}$$

by induction.