

Group A: Give a *combinatorial proof* to the identity

$$k \binom{n}{k} = n \binom{n-1}{k-1}$$

(Hint: Mark n positions with 0, 1, and 2. Or, see P154 #10.)

Group B: Use induction to prove that the number of different lattice paths from $(0, 0)$ to (m, n) is

$$\binom{m+n}{n}$$

Group C: Give a *combinatorial proof* to the identity

$$\binom{n}{k} = \binom{0}{k-1} + \binom{1}{k-1} + \cdots + \binom{n-2}{k-1} + \binom{n-1}{k-1}$$

(Hint: Consider the largest number when choosing k numbers out of n numbers)